

# Quantifying Information Leakage of Deterministic Encryption

Mireya Jurado and Geoffrey Smith

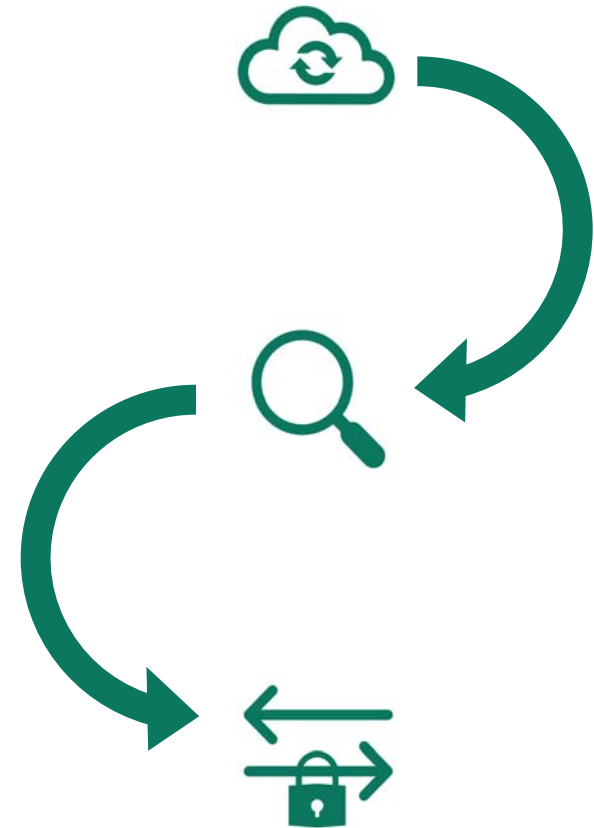
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CCSW 2019: Cloud Computing Security Workshop



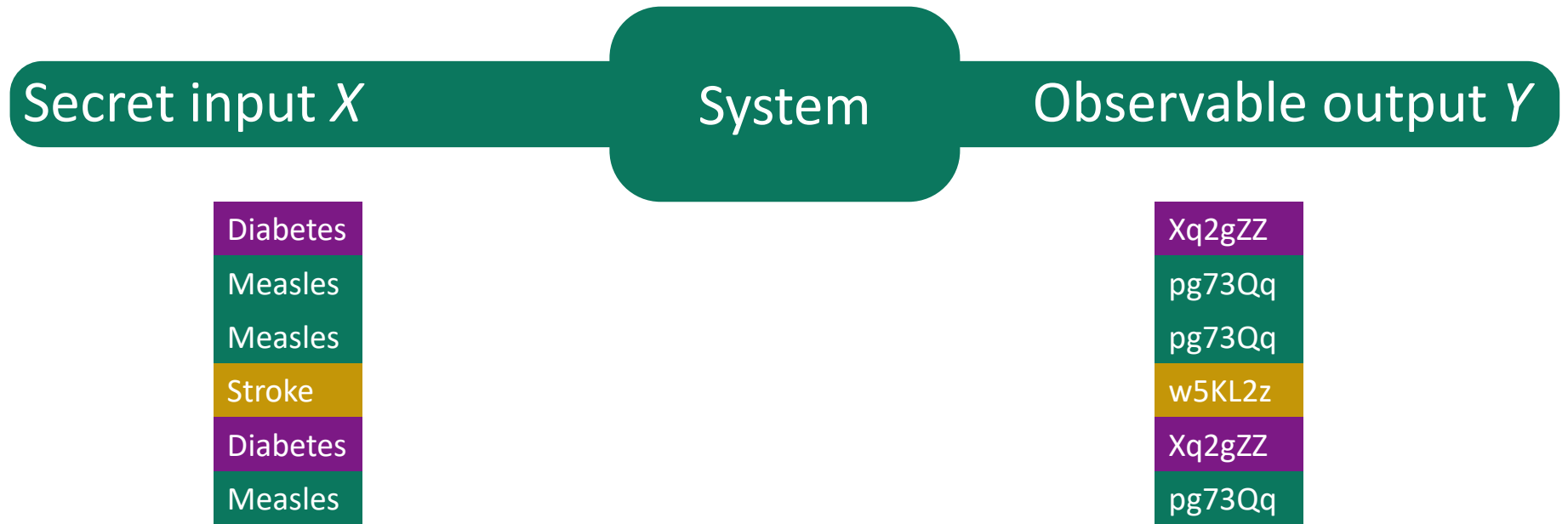
# Challenge

- We want to store sensitive data remotely
- Encrypted databases aim to balance security and functionality



Question: How do you measure information leakage of deterministic encryption?

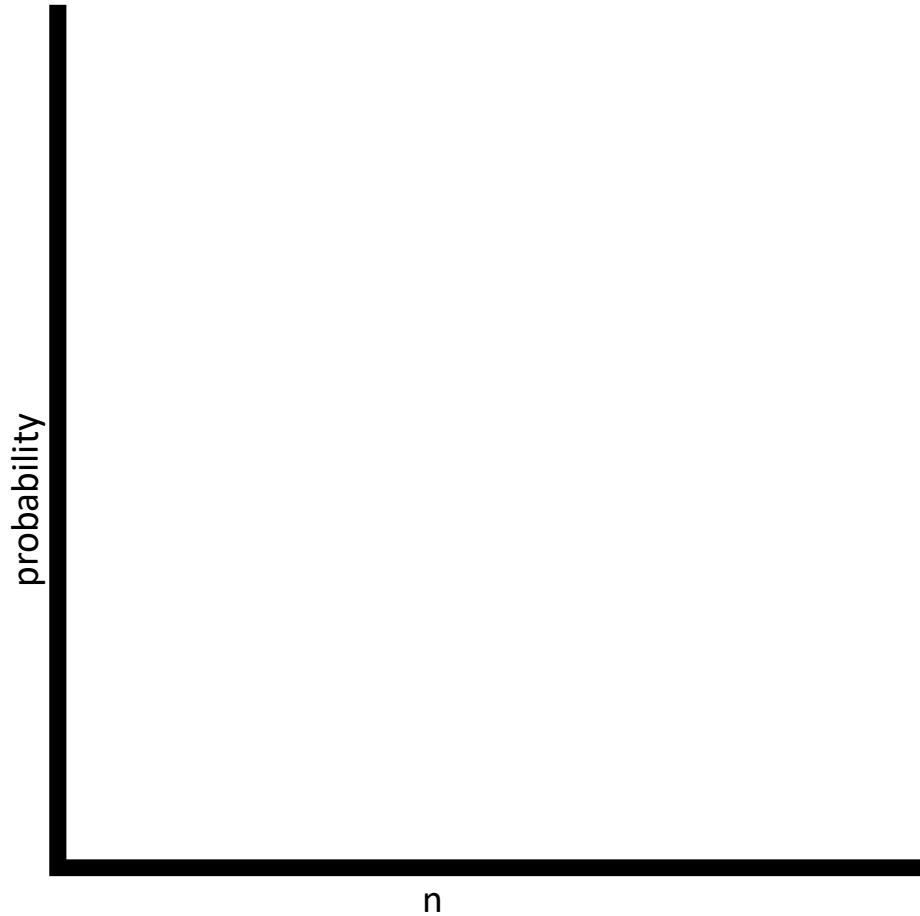
# Quantitative Information Flow (QIF)



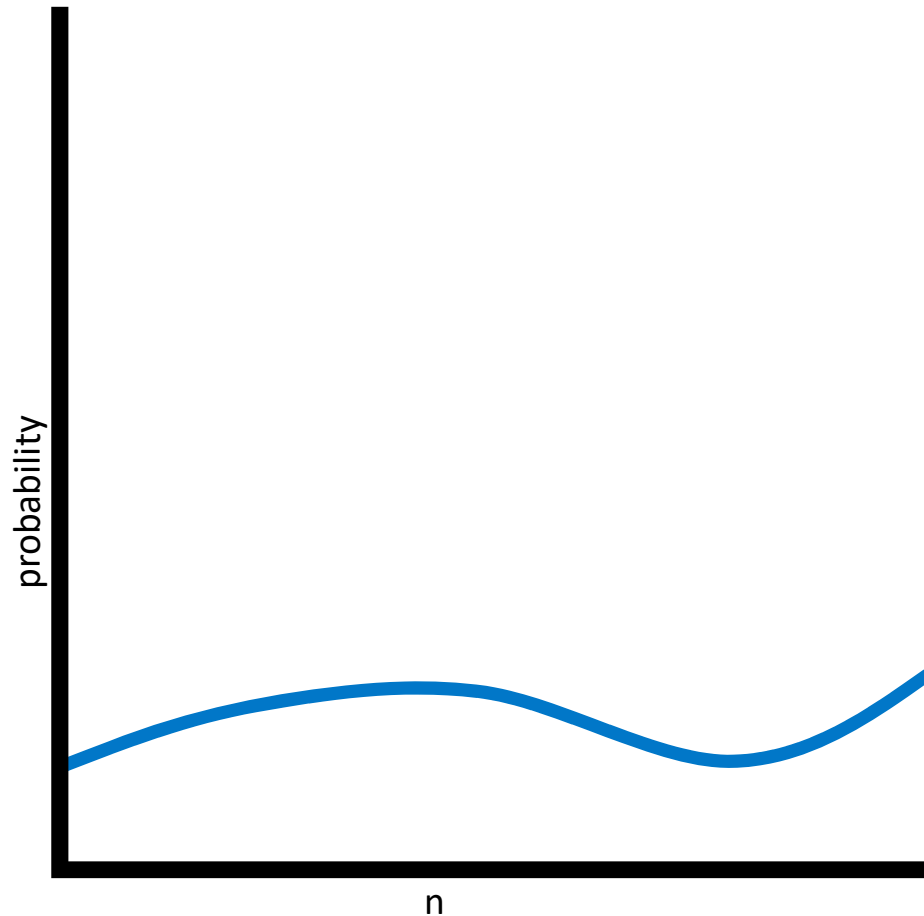
The database column of diseases, drawn independently according to some prior distribution  $\delta$  on diseases

The deterministic encryption of the column, modeled as a random permutation (the “ideal object”)

# Quantitative Information Flow Concepts

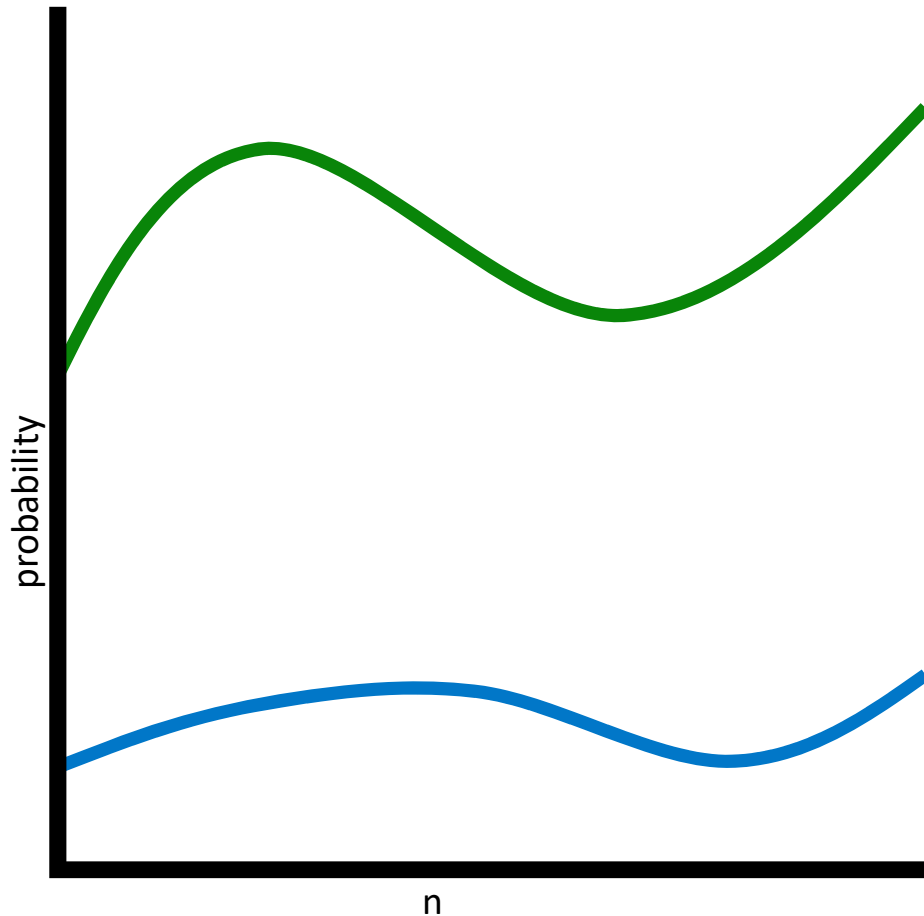


# Quantitative Information Flow Concepts



**Prior vulnerability:**  
Adversary's probability of  
accomplishing goal given only  $\delta$

# Quantitative Information Flow Concepts



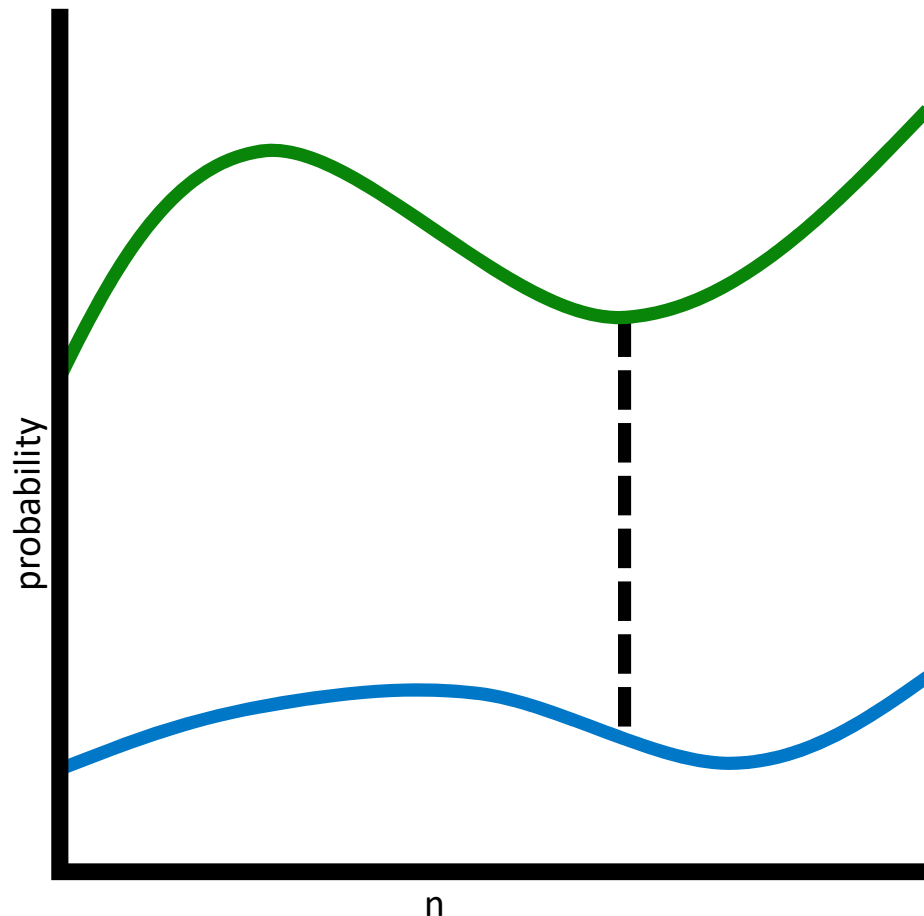
## **Posterior vulnerability:**

Adversary's probability of accomplishing goal given  $\delta$  and output  $Y$

## **Prior vulnerability:**

Adversary's probability of accomplishing goal given only  $\delta$

# Quantitative Information Flow Concepts



## **Posterior vulnerability:**

Adversary's probability of accomplishing goal given  $\delta$  and output  $Y$

## **Leakage:**

The difference between prior and posterior vulnerability

## **Prior vulnerability:**

Adversary's probability of accomplishing goal given only  $\delta$

n

# Intuition of Model

Disease	Probability $\delta$
a	$1/2$
b	$1/3$
c	$1/6$

Xq2gZZ  
pg73Qq  
pg73Qq  
w5KL2z  
Xq2gZZ  
pg73Qq



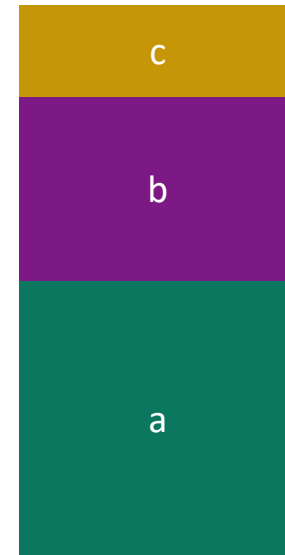
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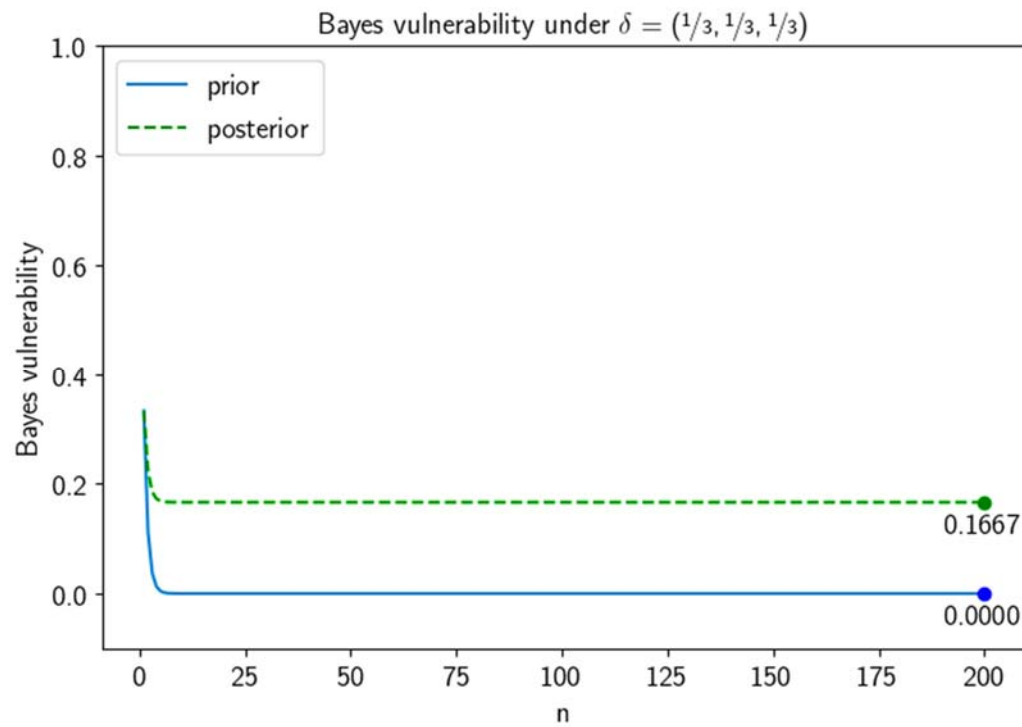


# Results: Bayes Scenario

Goal: guess the entire secret in one try

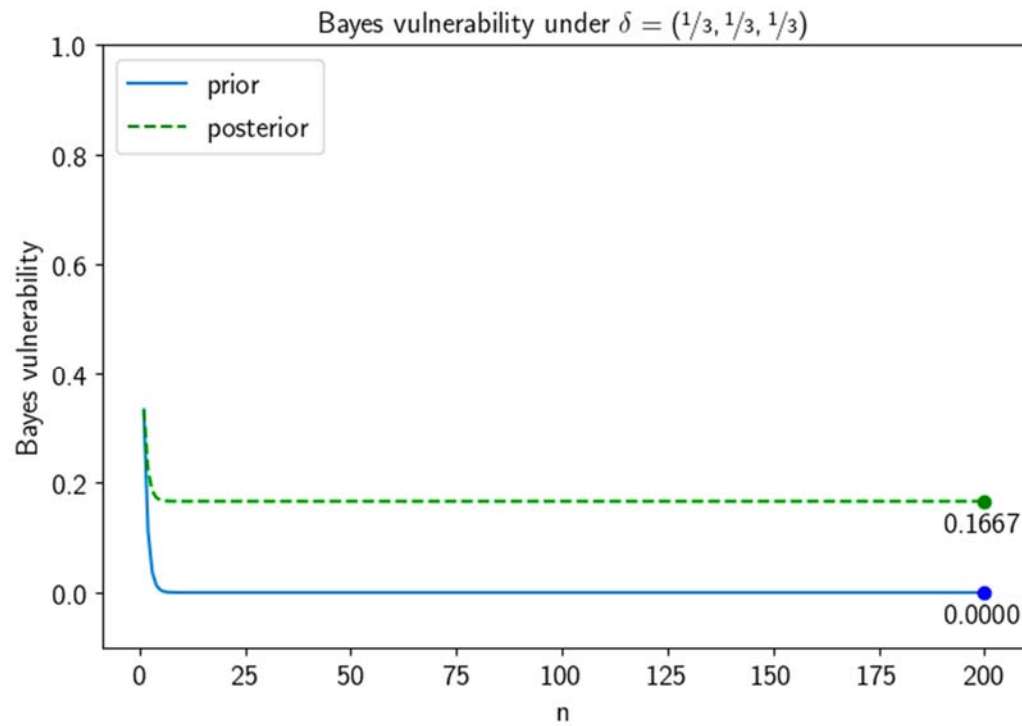
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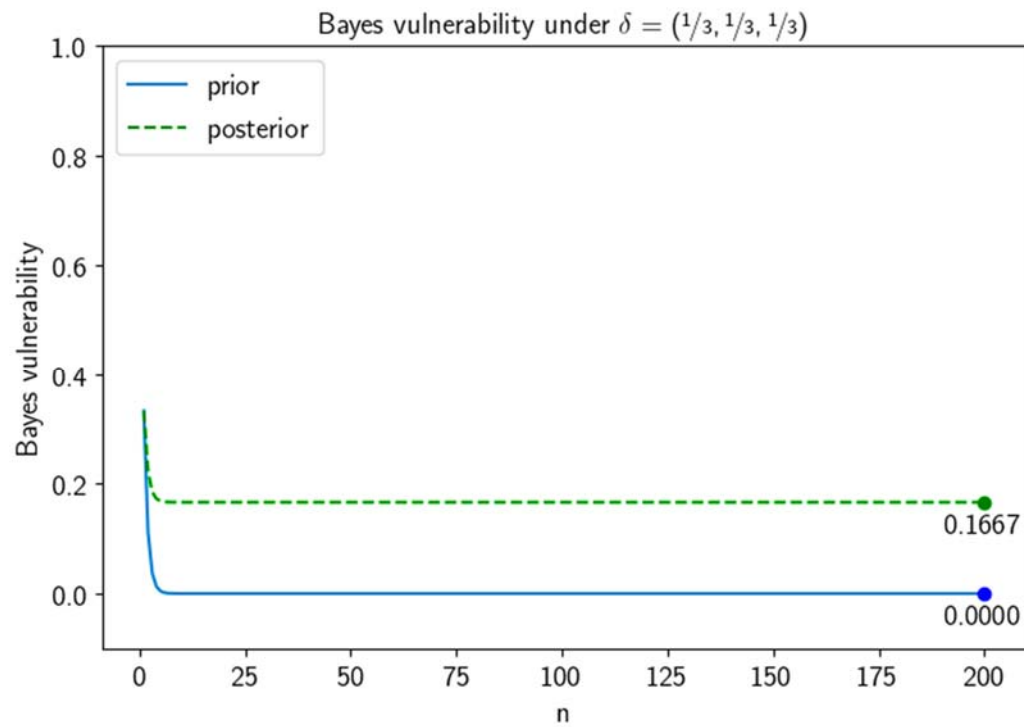
Prior Vulnerability:

When n = 1:

$1/3$

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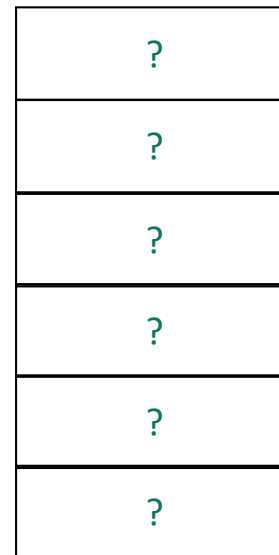
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$$1/3$$

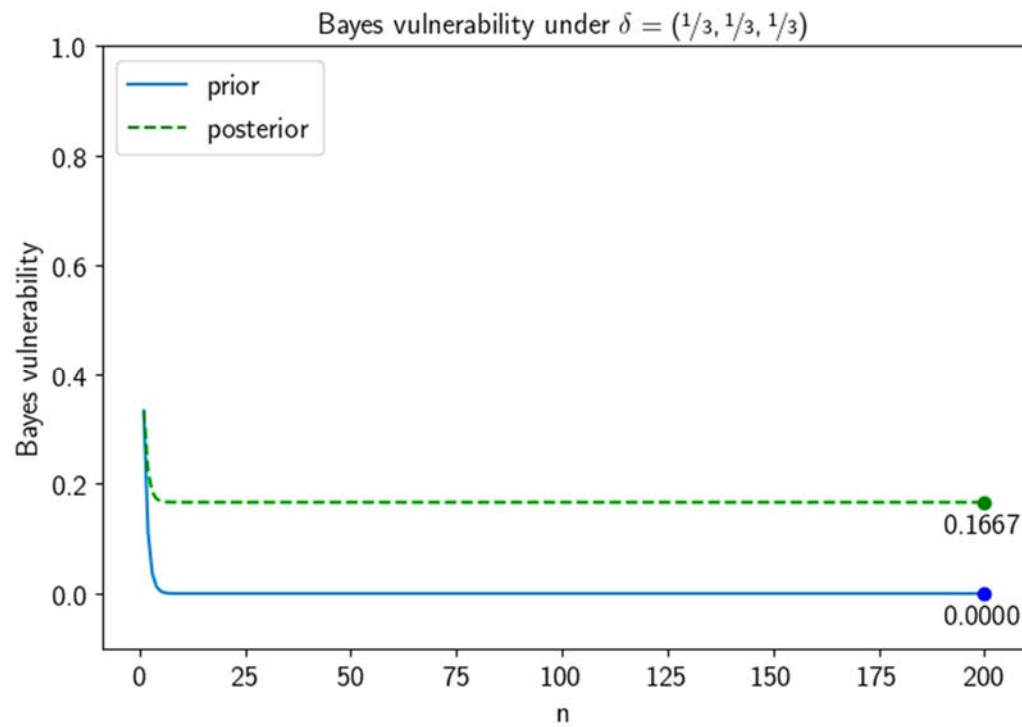
When n is large:



$$1/3^n$$

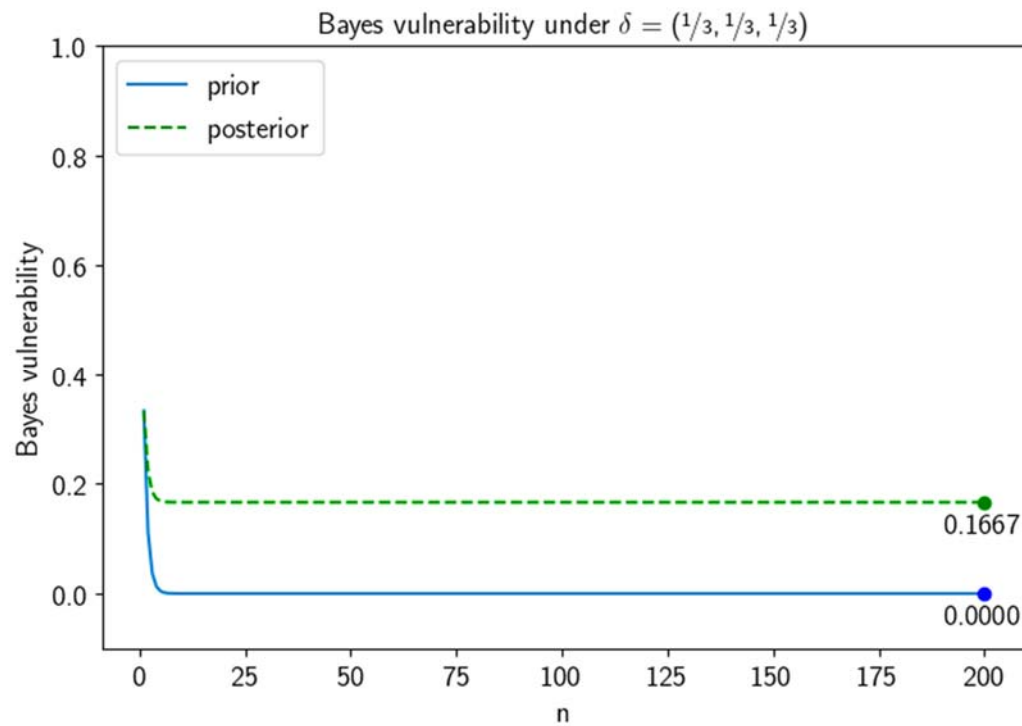
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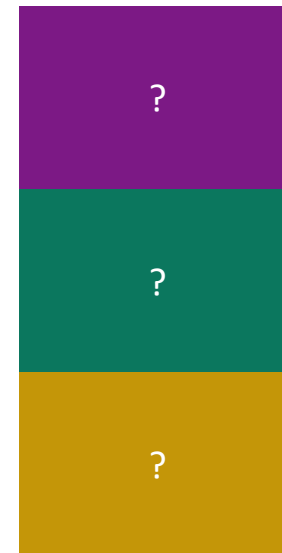
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Posterior Vulnerability:

When n is large:

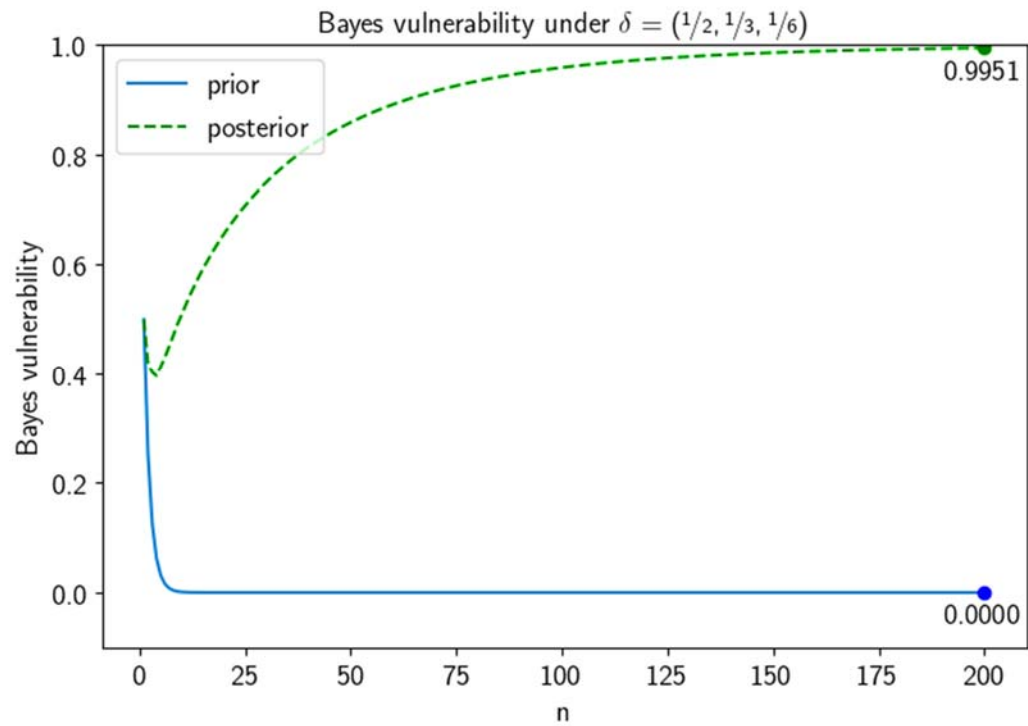


$$1/6$$



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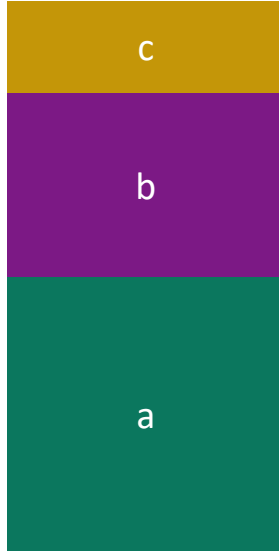


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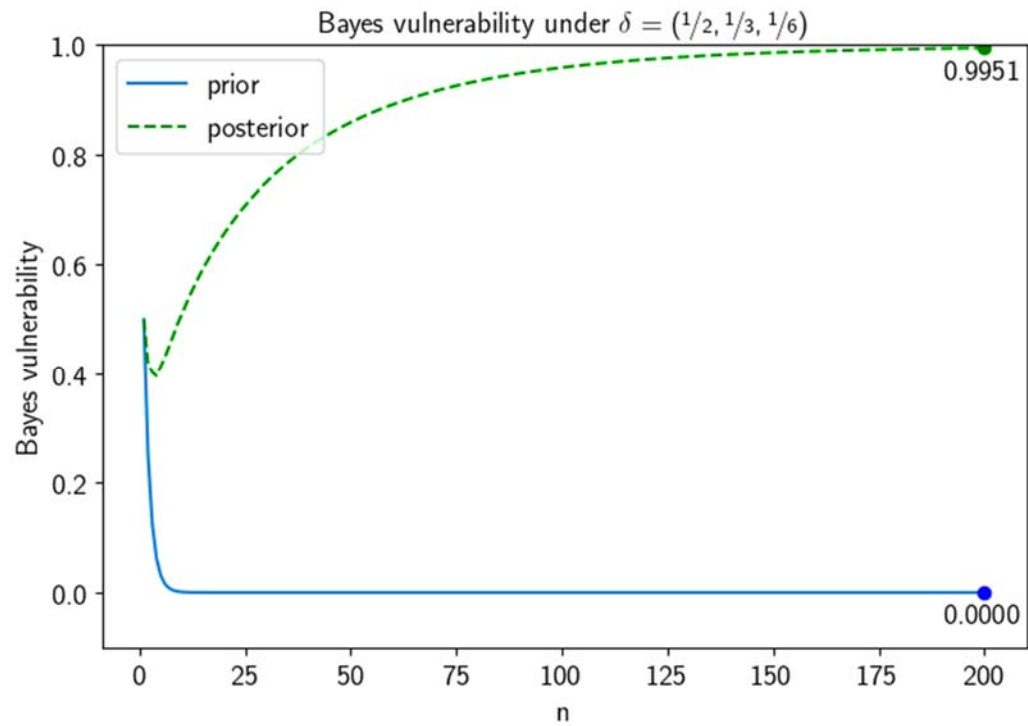
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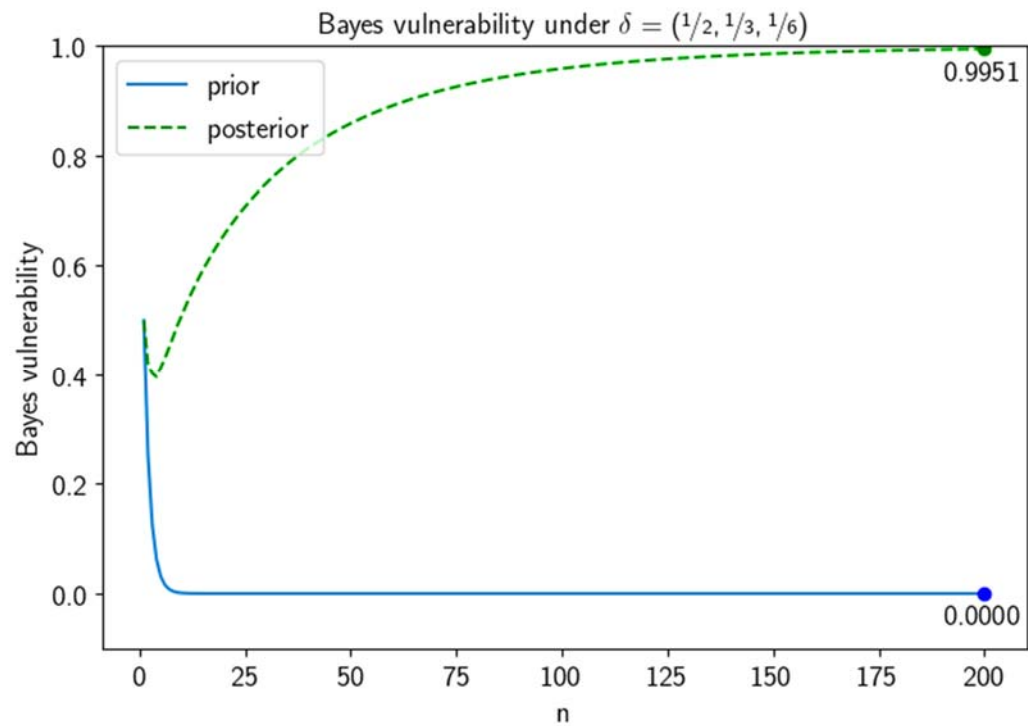
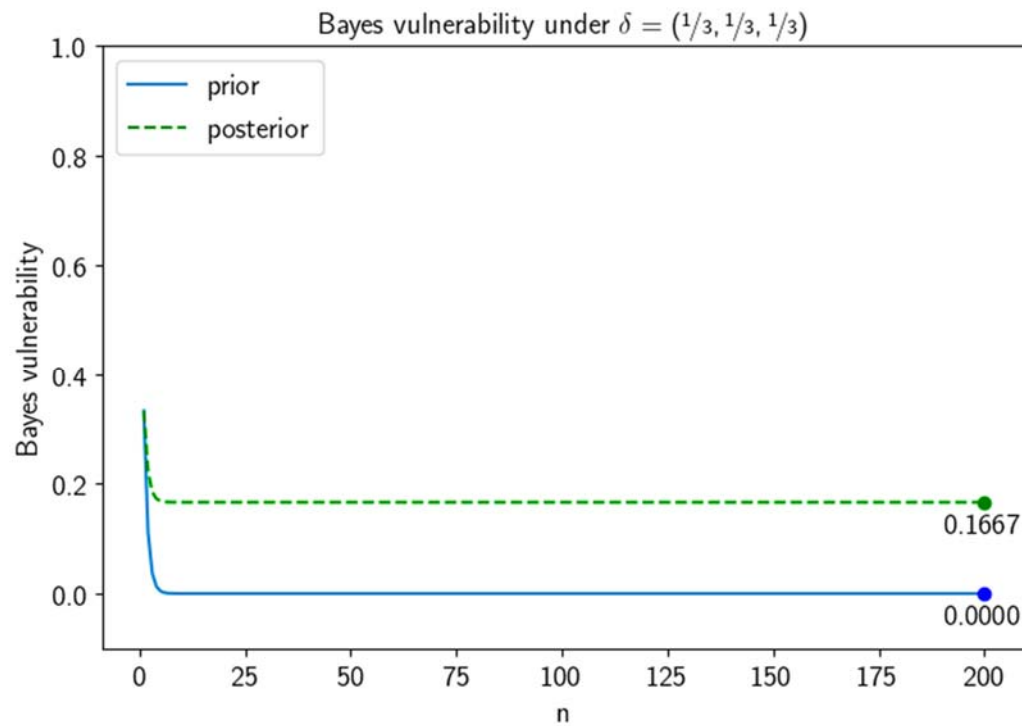


grows to 1



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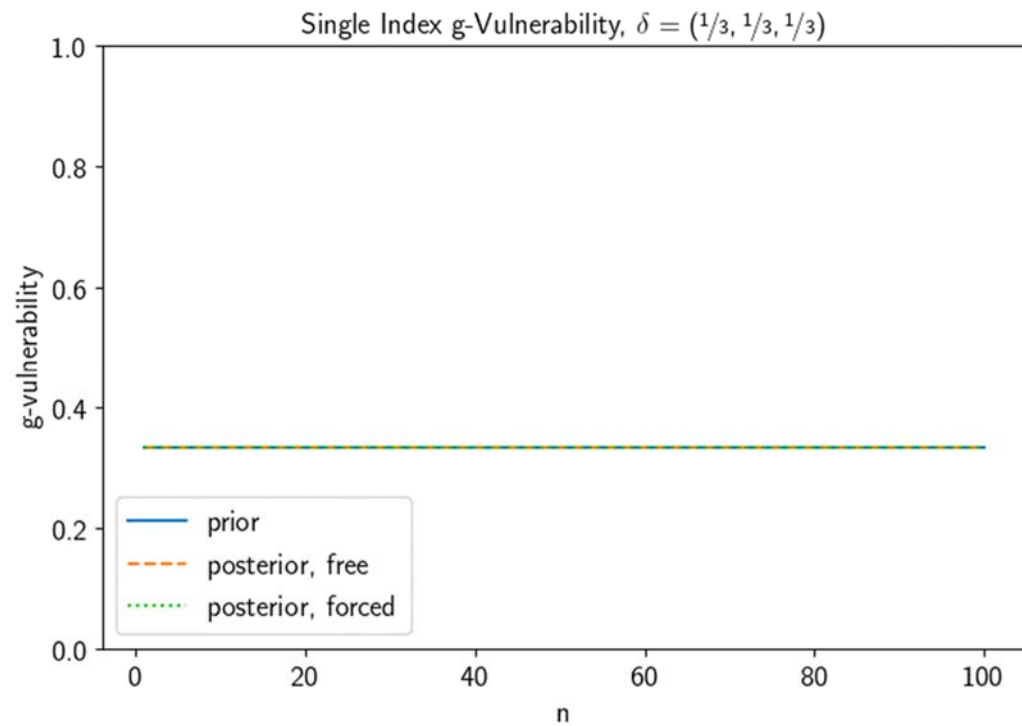


# Results: Single Index Scenarios

Goals: (1) *free* to guess any patient's disease and (2) *forced* to guess a specified patient's disease

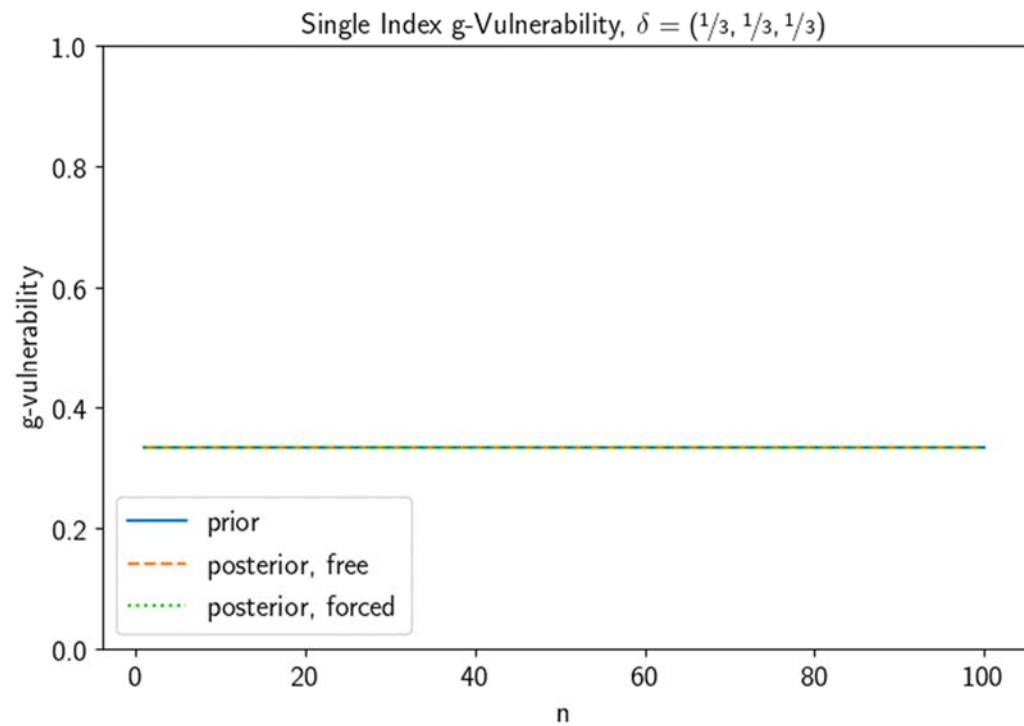
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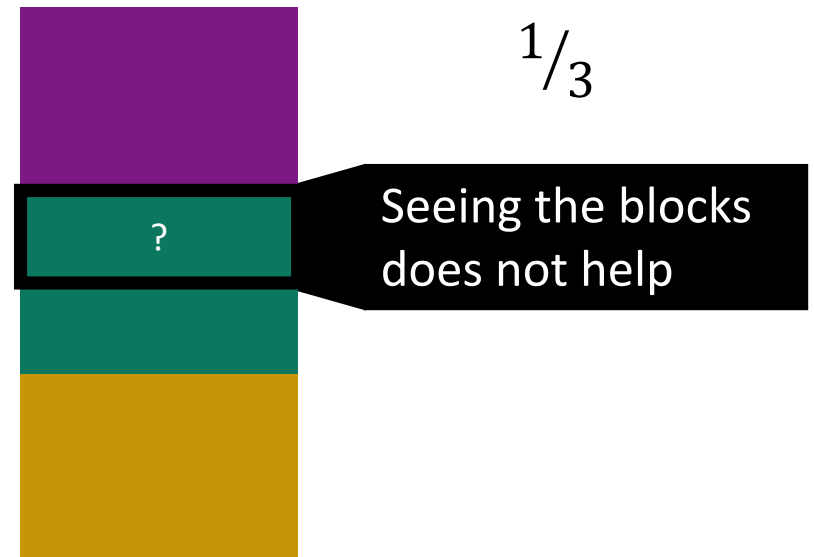


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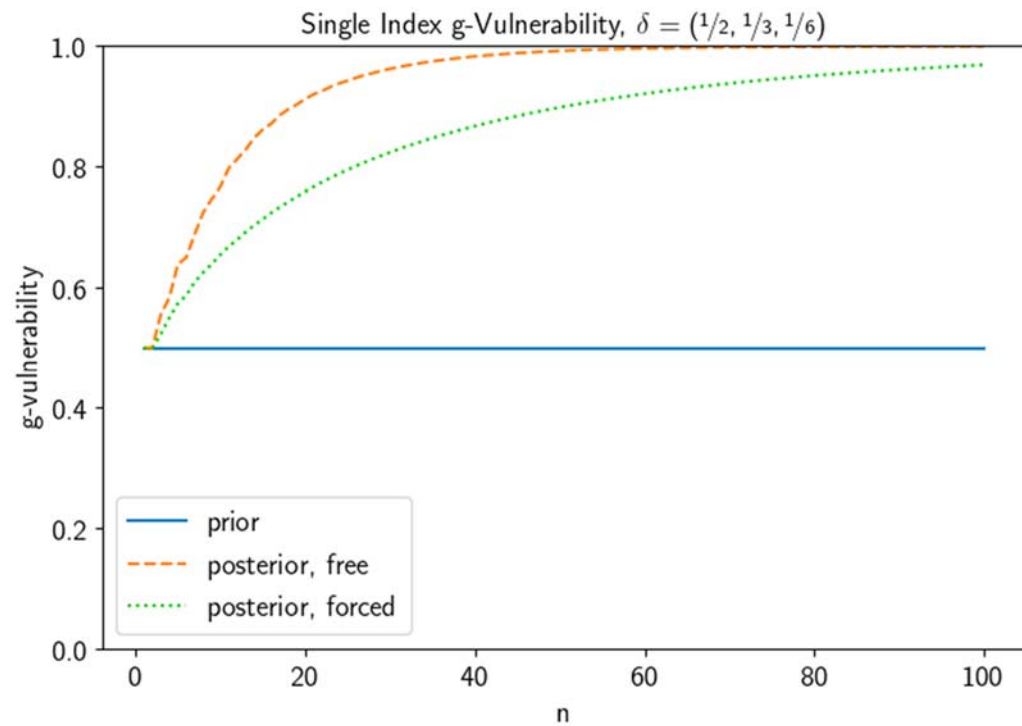


Posterior Vulnerability:



# Results: Single Index Scenarios

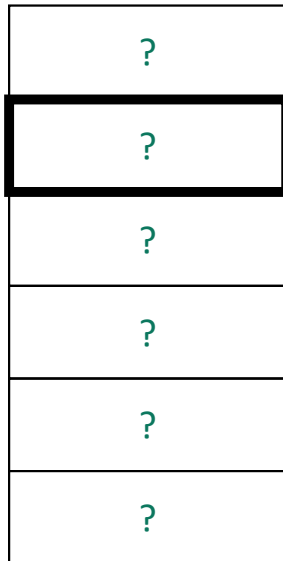
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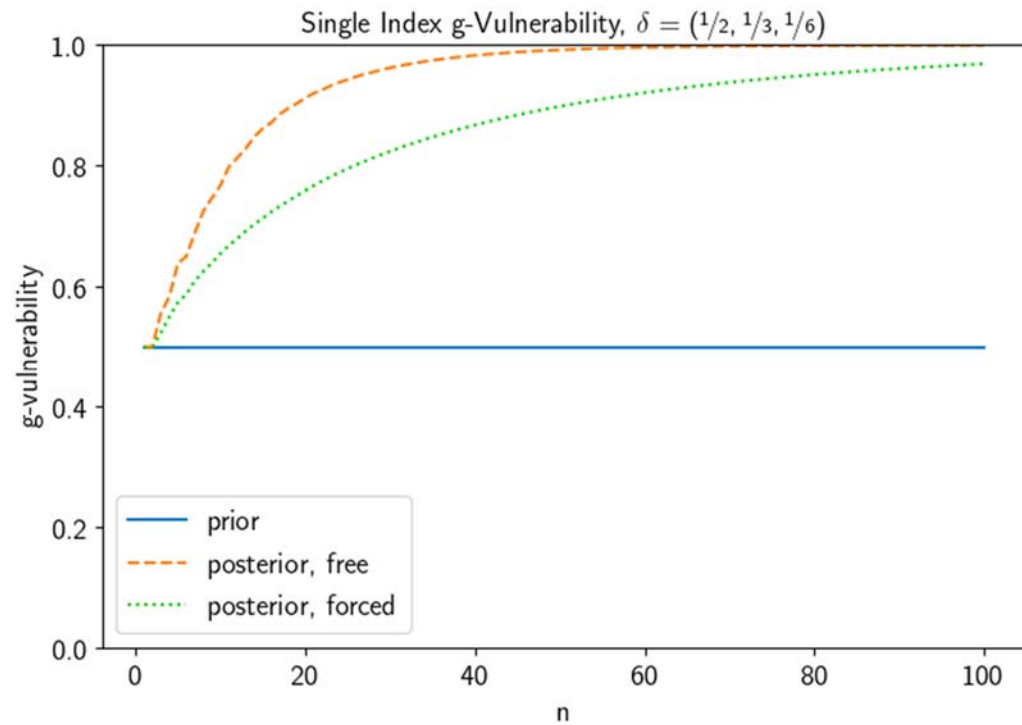
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Prior Vulnerability:



$1/2$

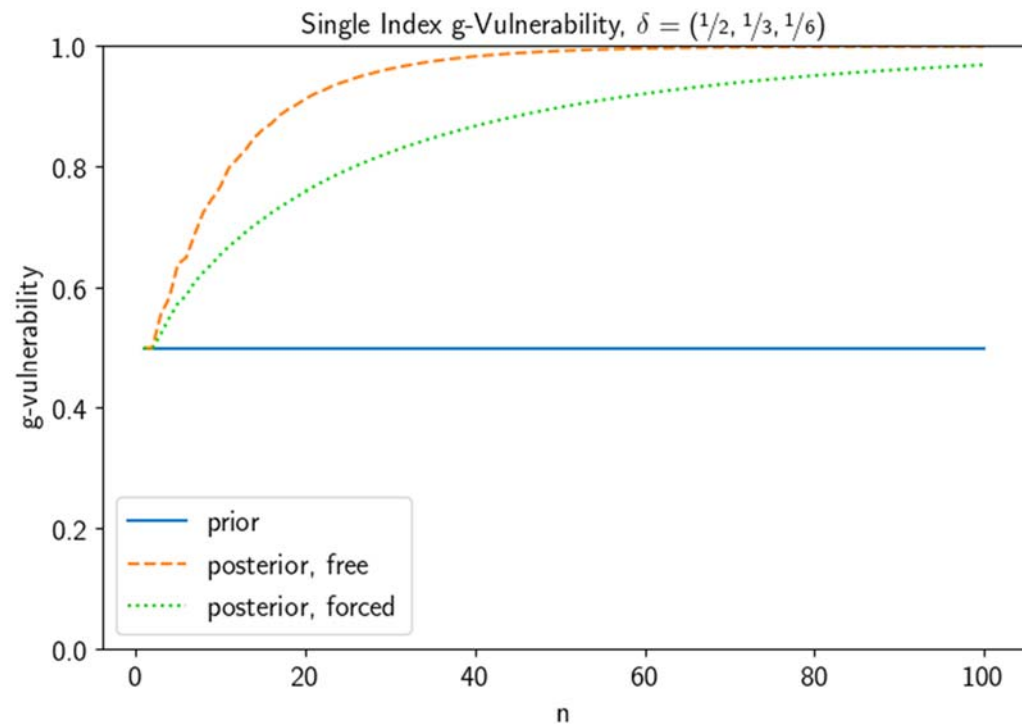




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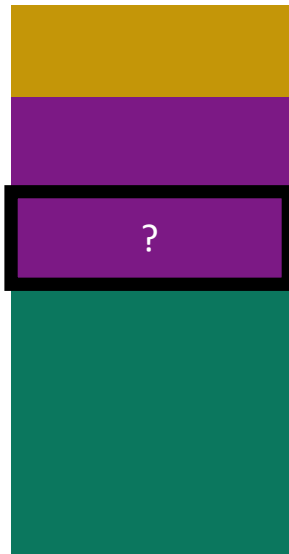
Posterior Vulnerability (free):



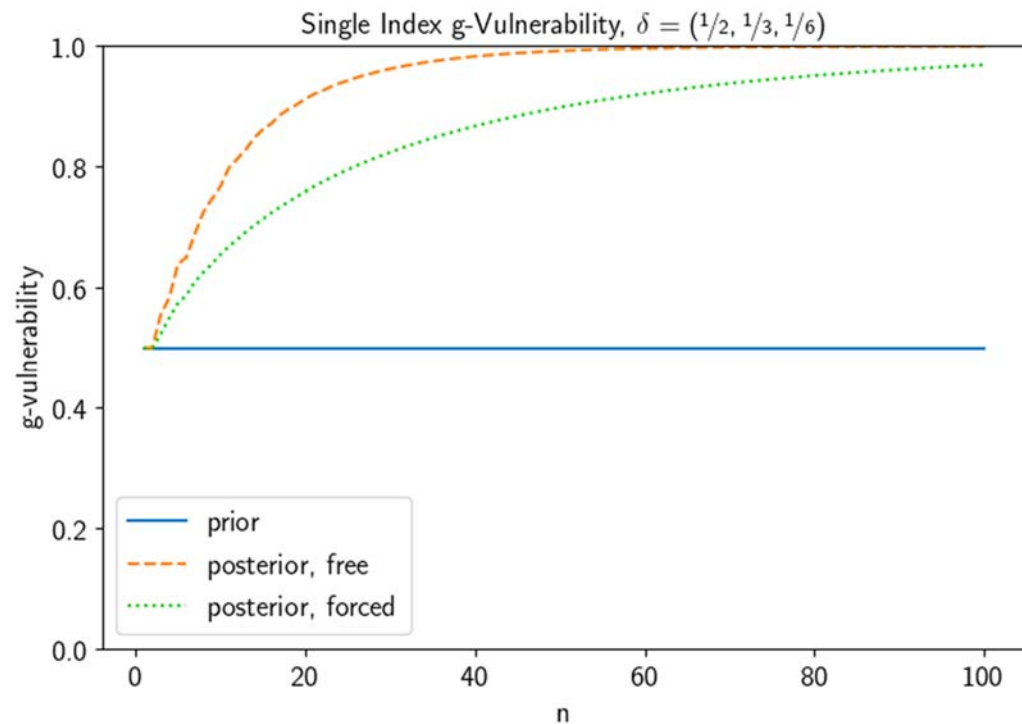
# Results: Single Index Scenarios

Goals: (1) *free* to guess any patient's disease and (2) *forced* to guess a specified patient's disease

Posterior Vulnerability (forced):

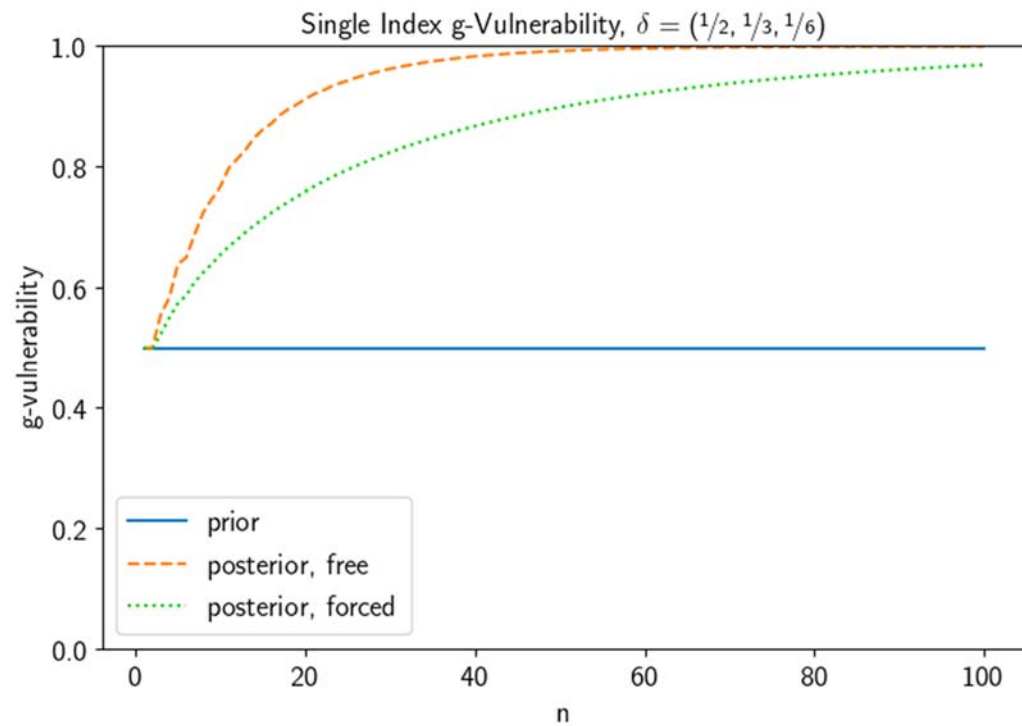
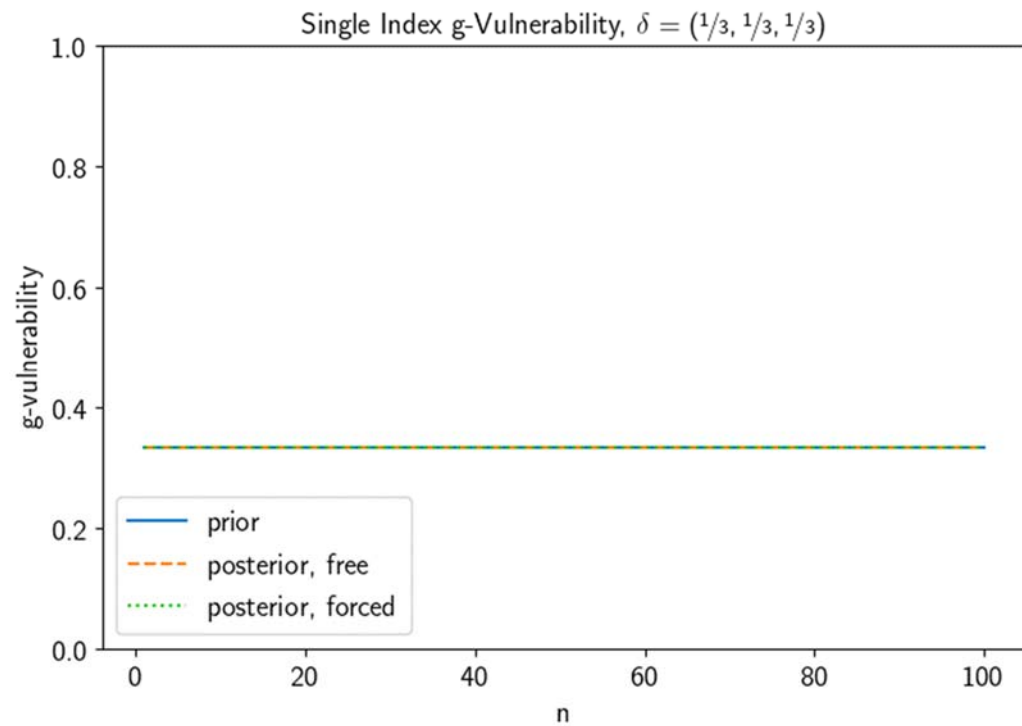


Grows slower

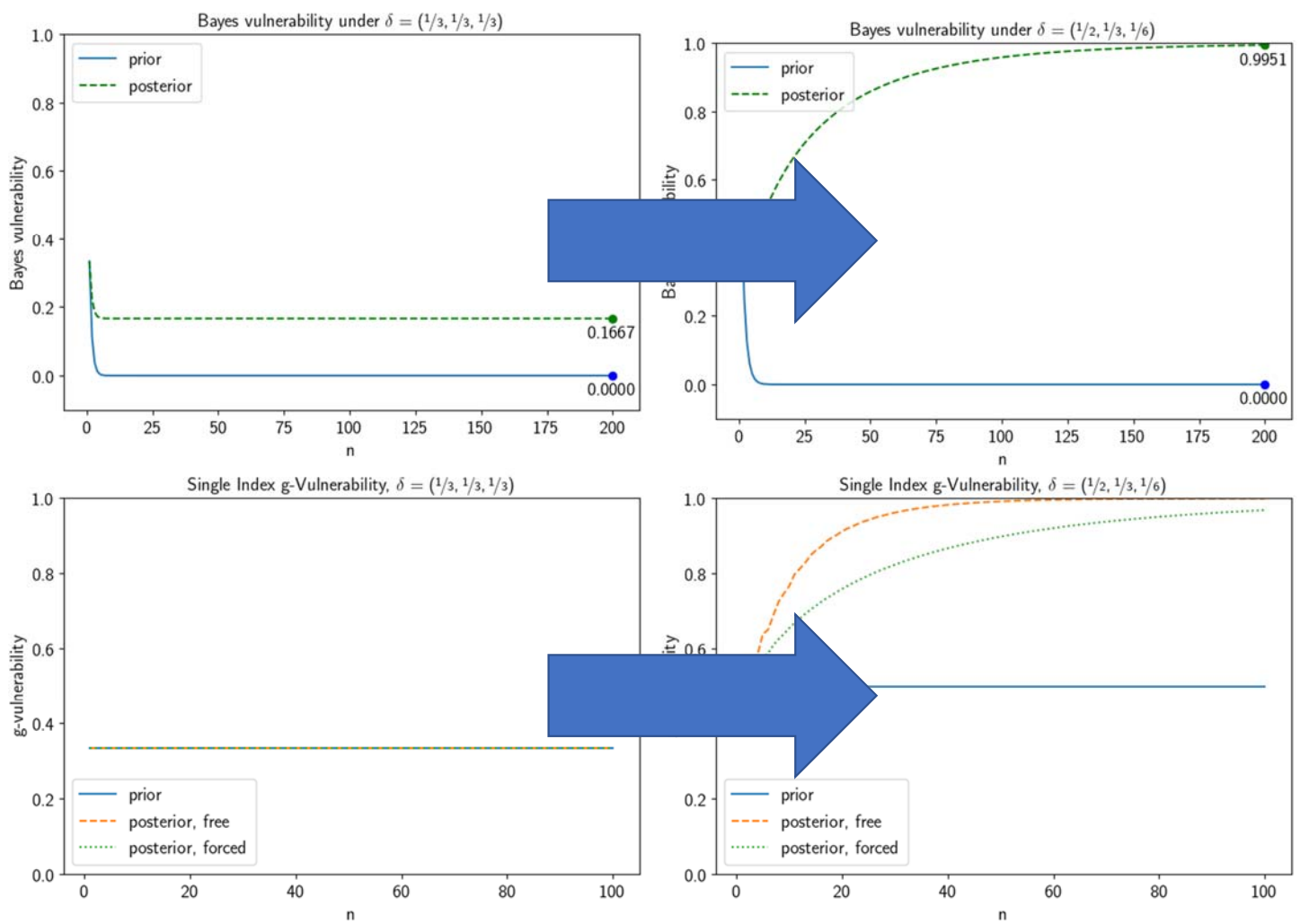


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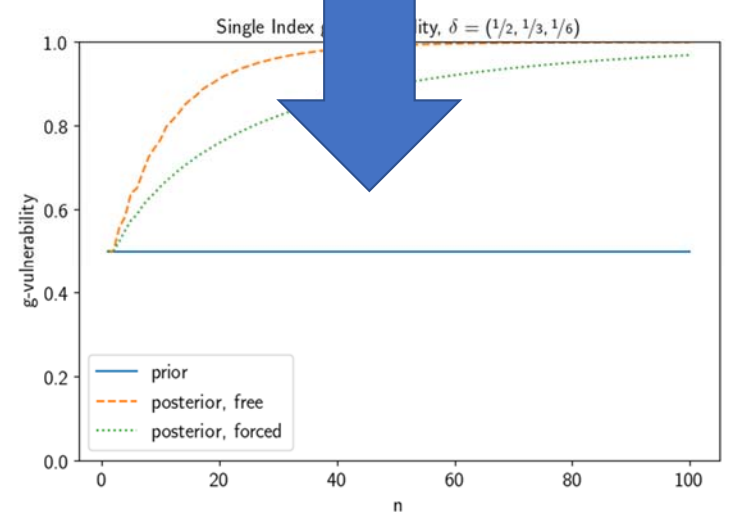
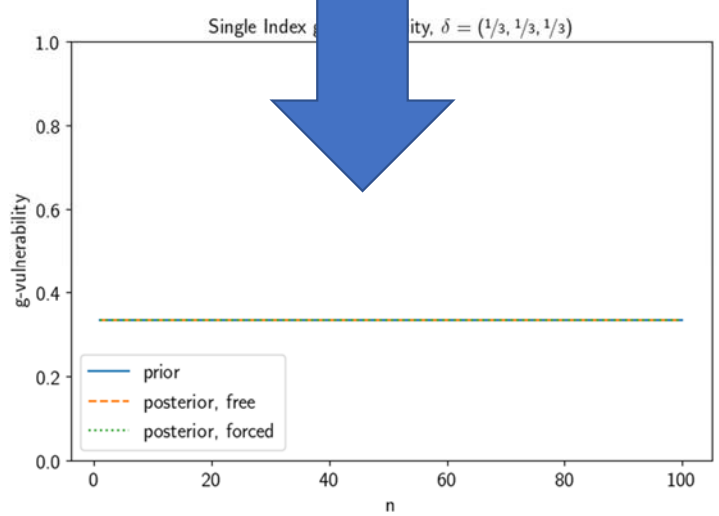
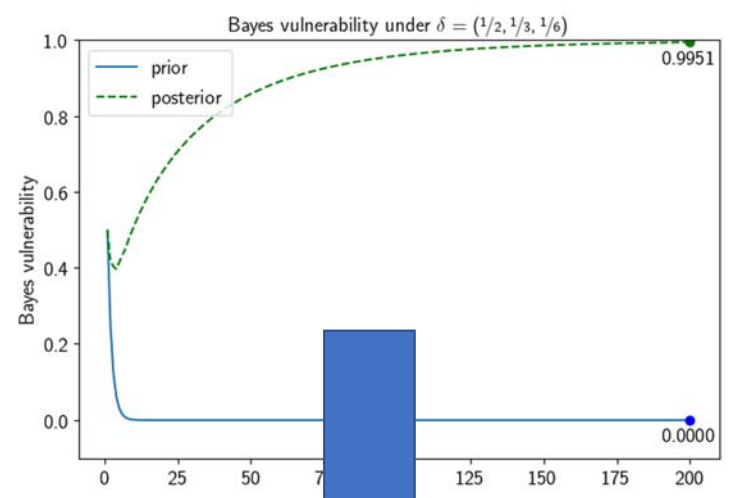
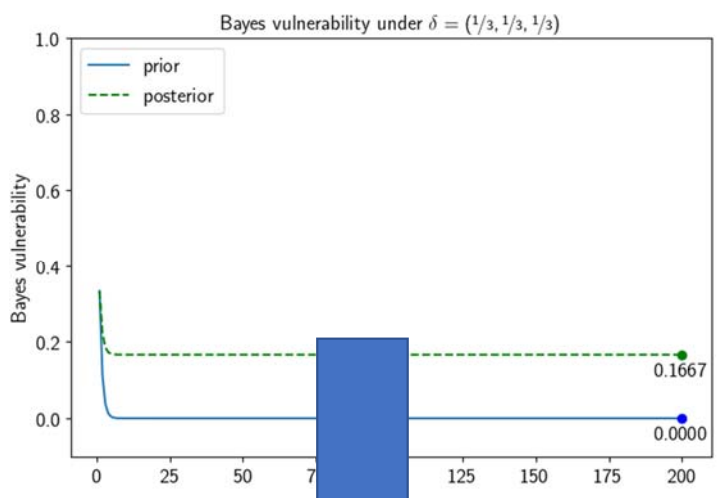
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# Leakage Depends on Prior and Operational Scenario

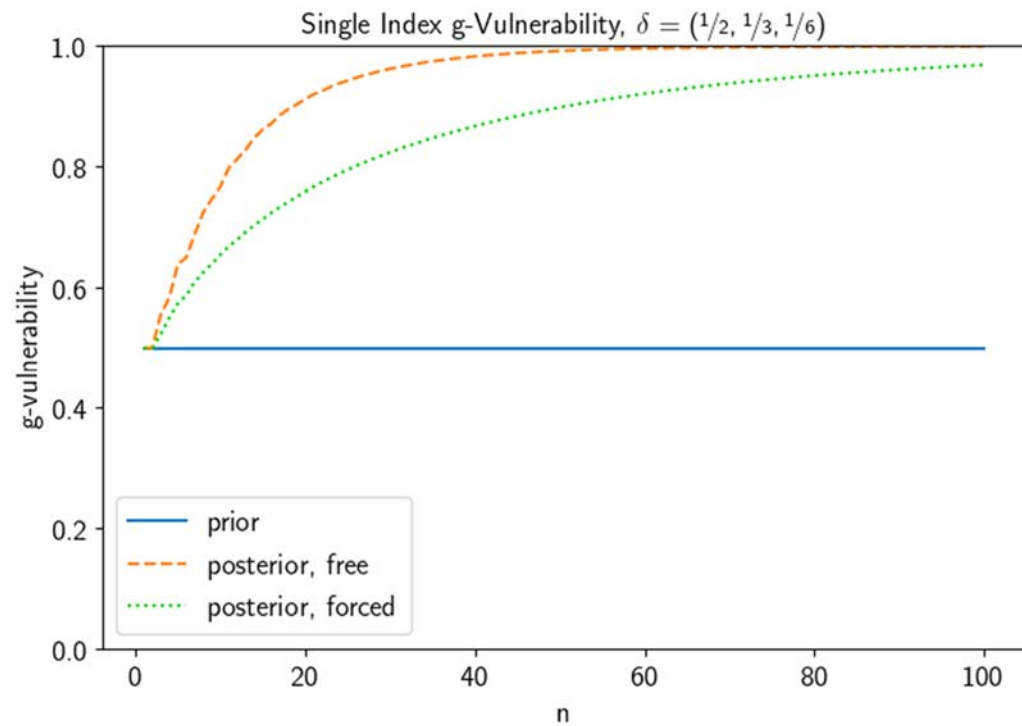
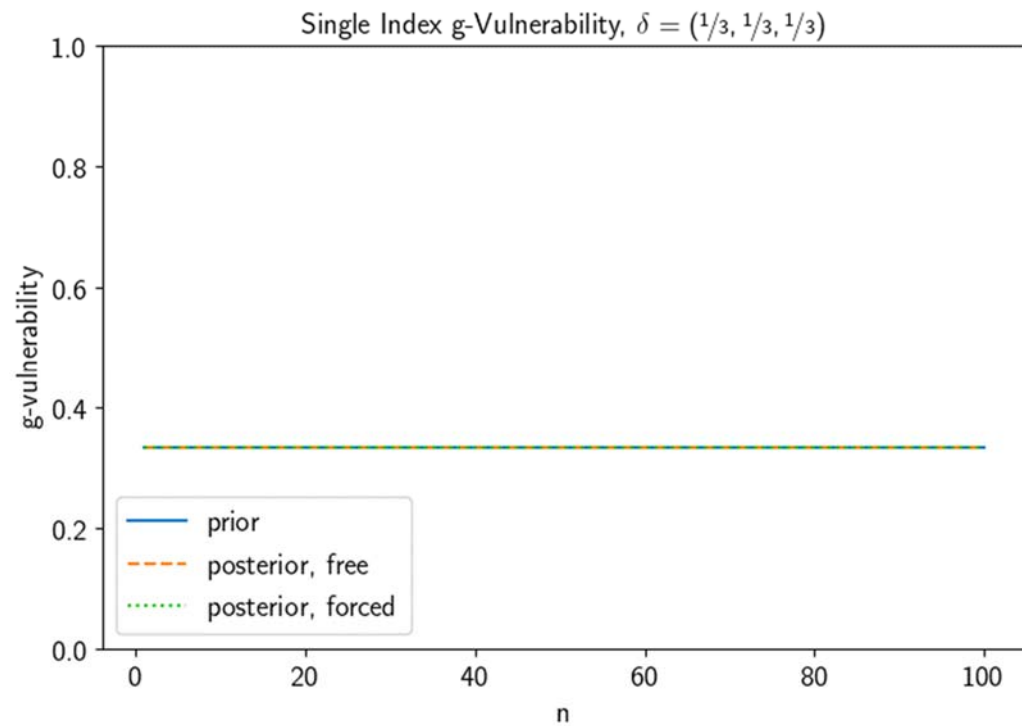


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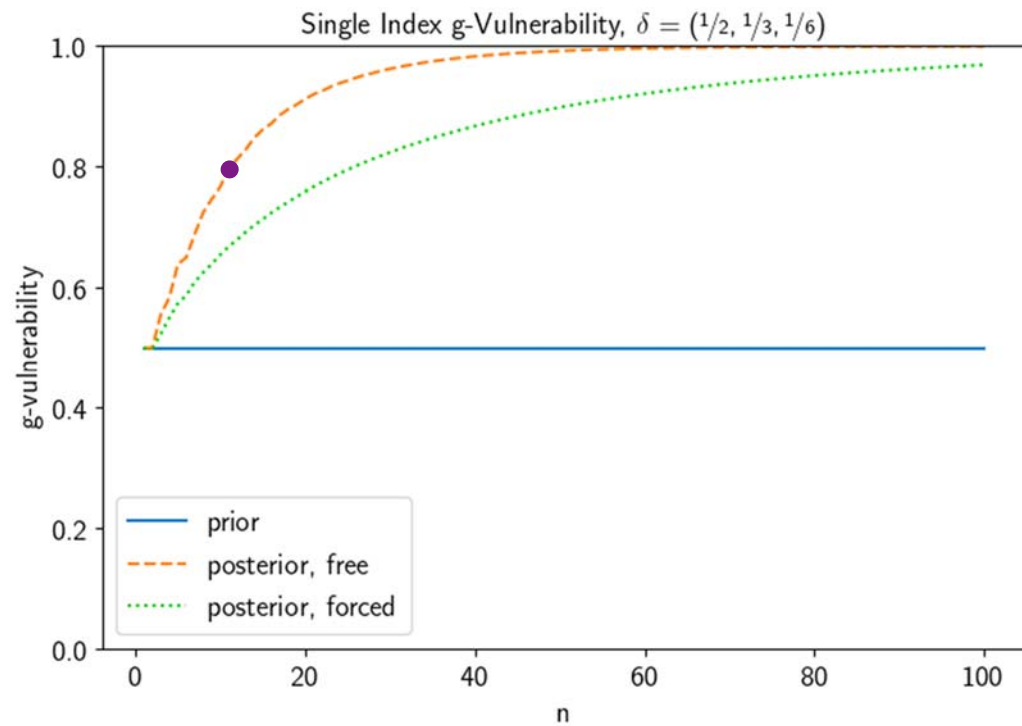
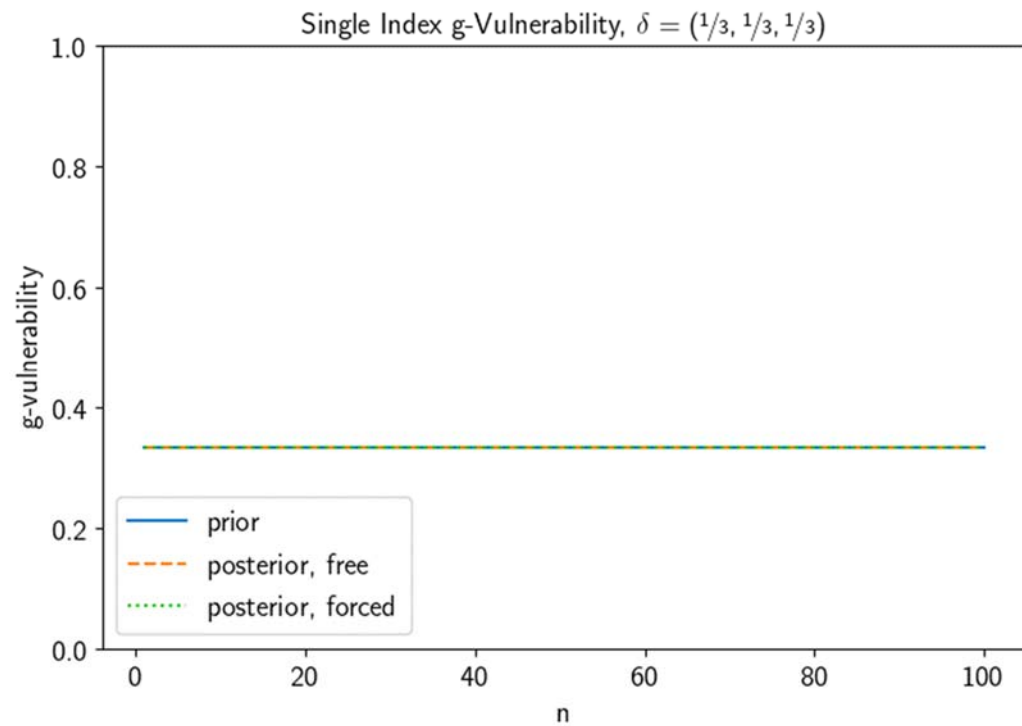
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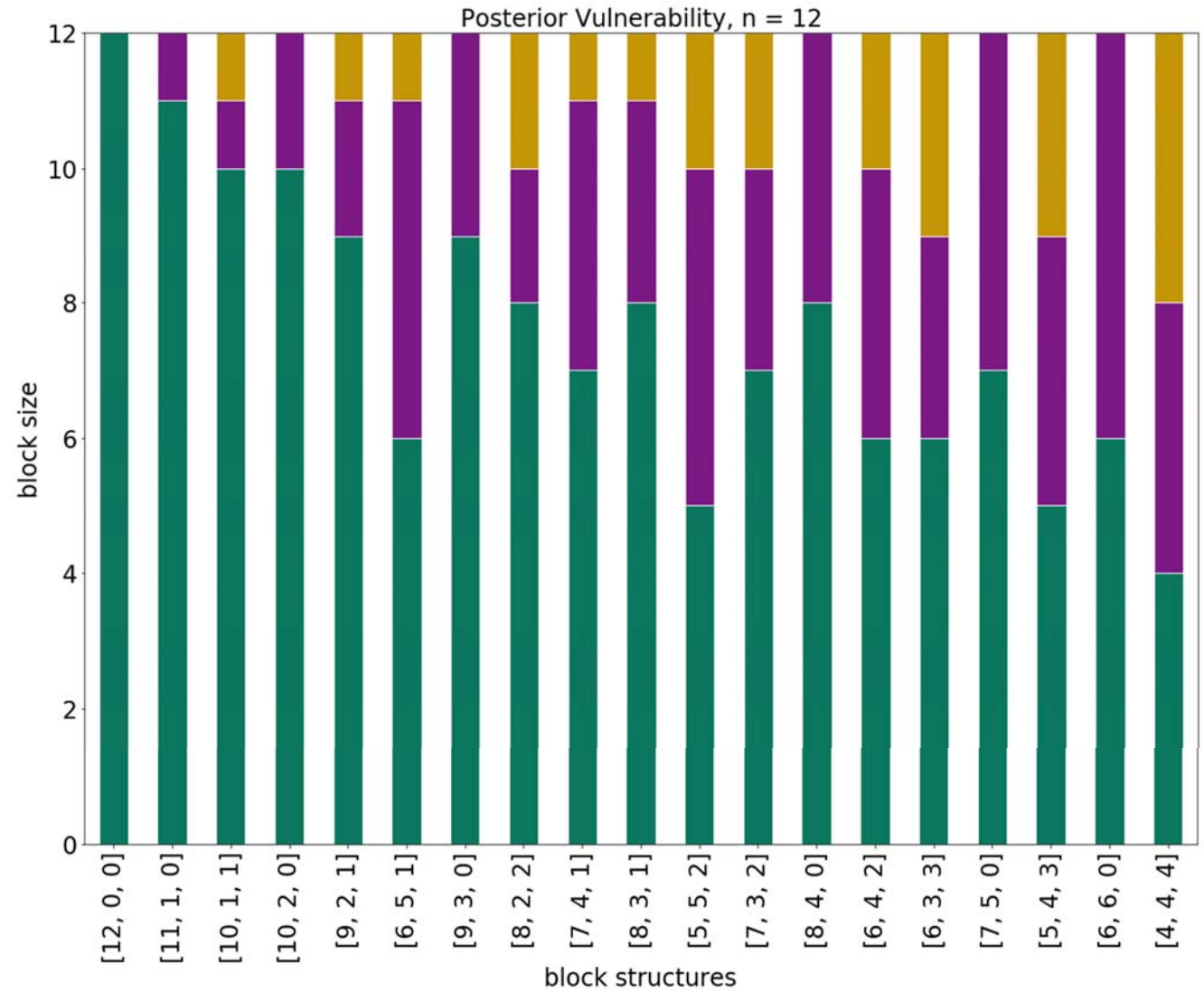
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# An In Depth View

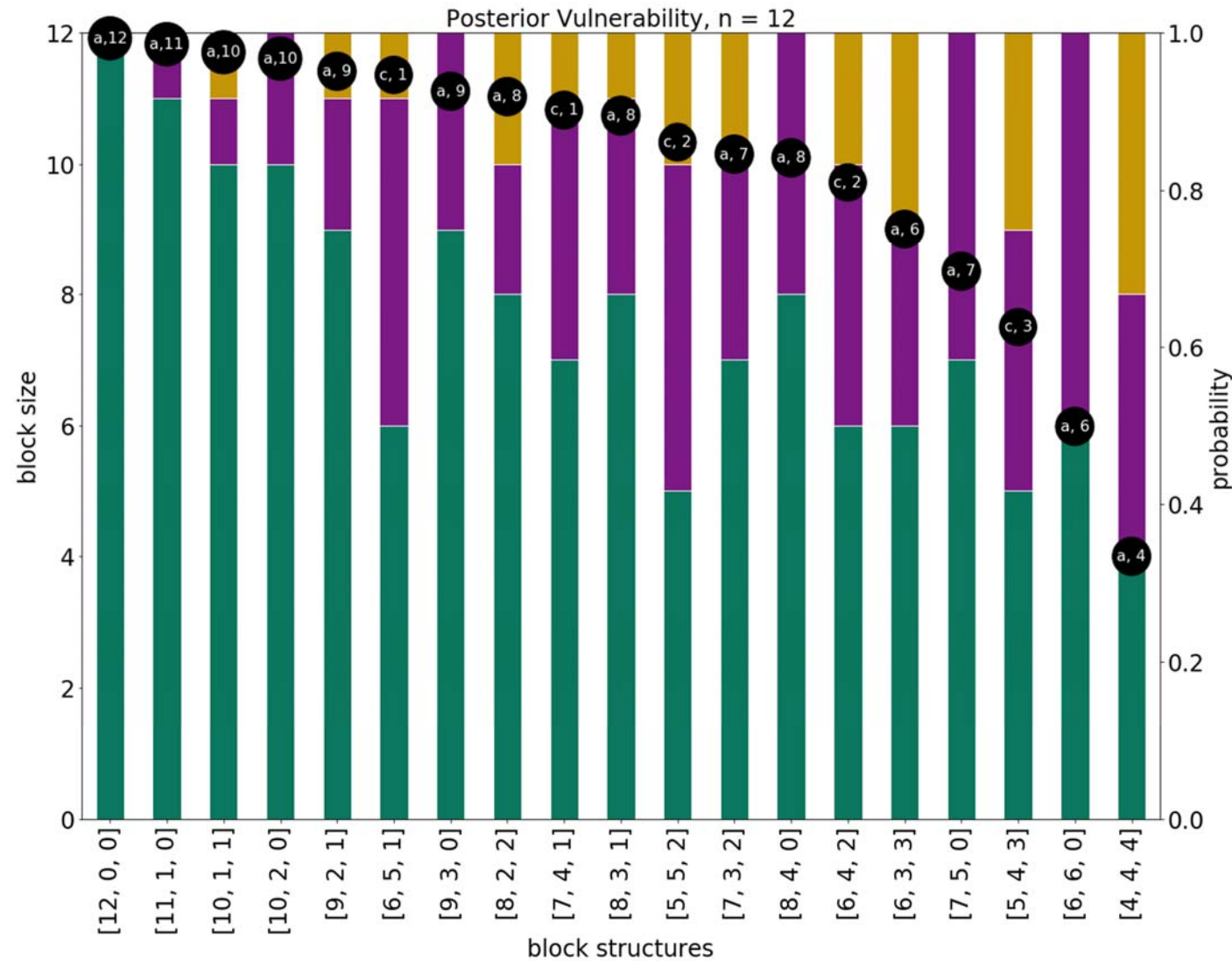
- 19 possible block structures





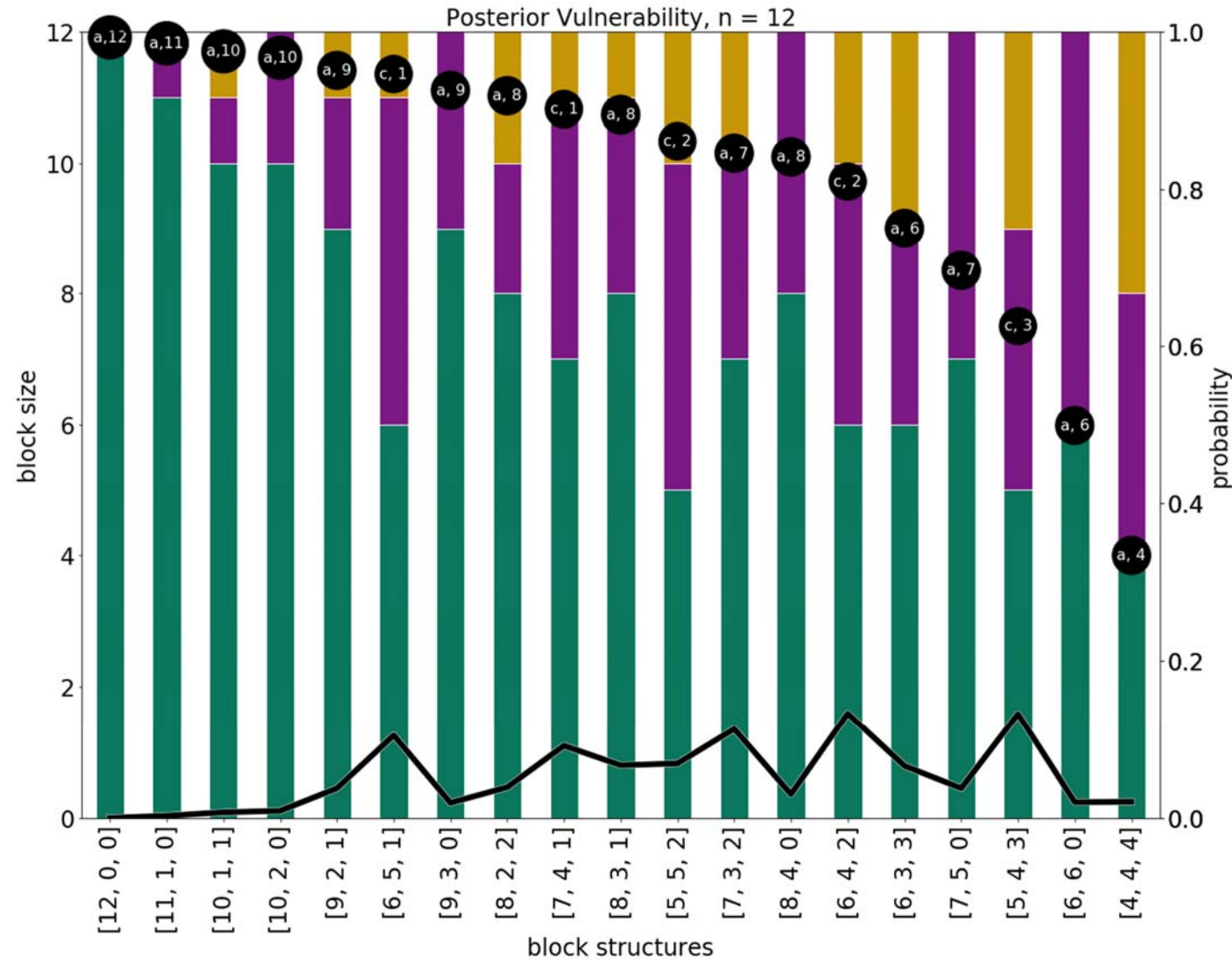
# An In Depth View

- 19 possible block structures
- Dots: the best guess and her probability of being correct



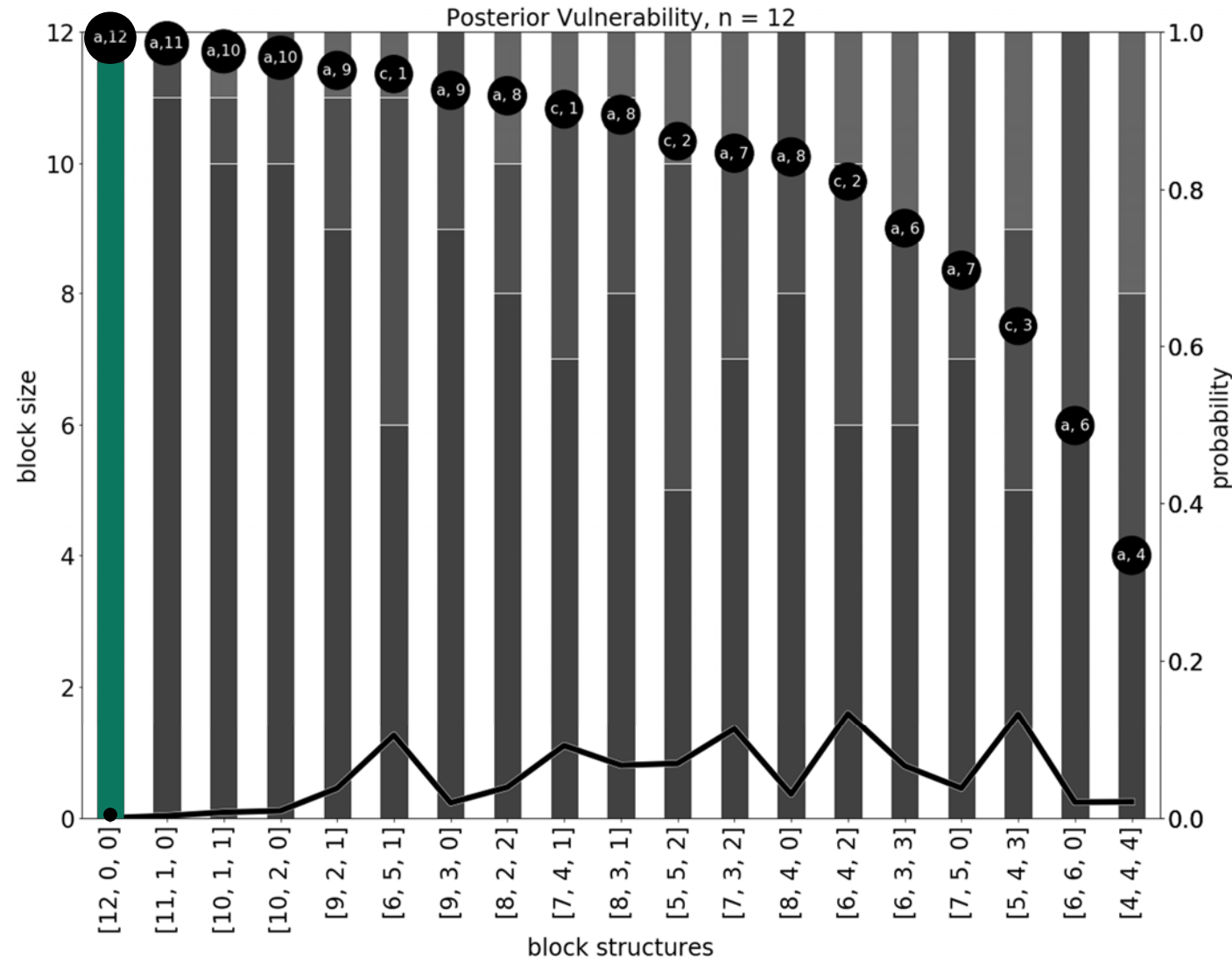
# An In Depth View

- 19 possible block structures
- Dots: the best guess and her probability of being correct
- Line: the probability the block structure will occur



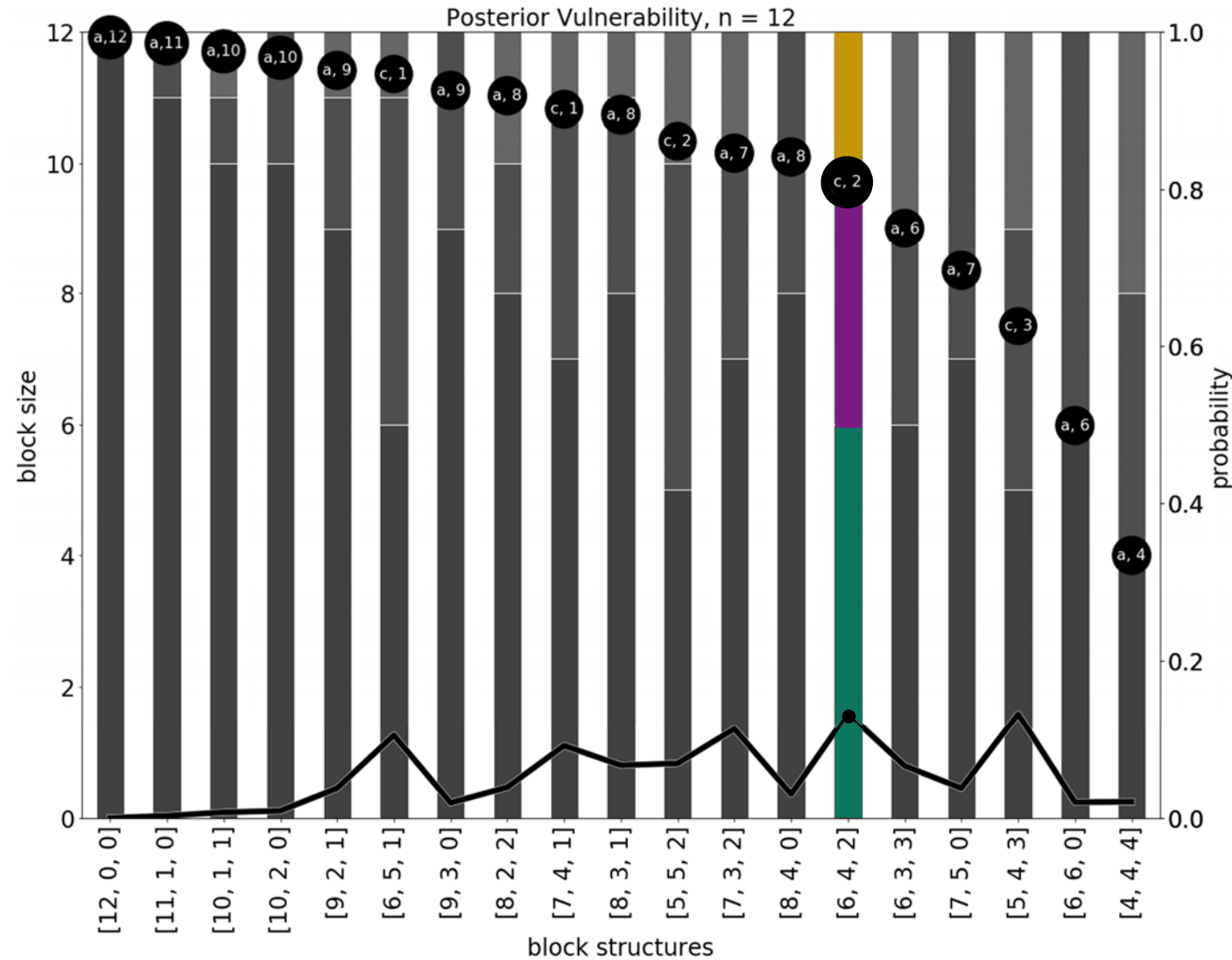
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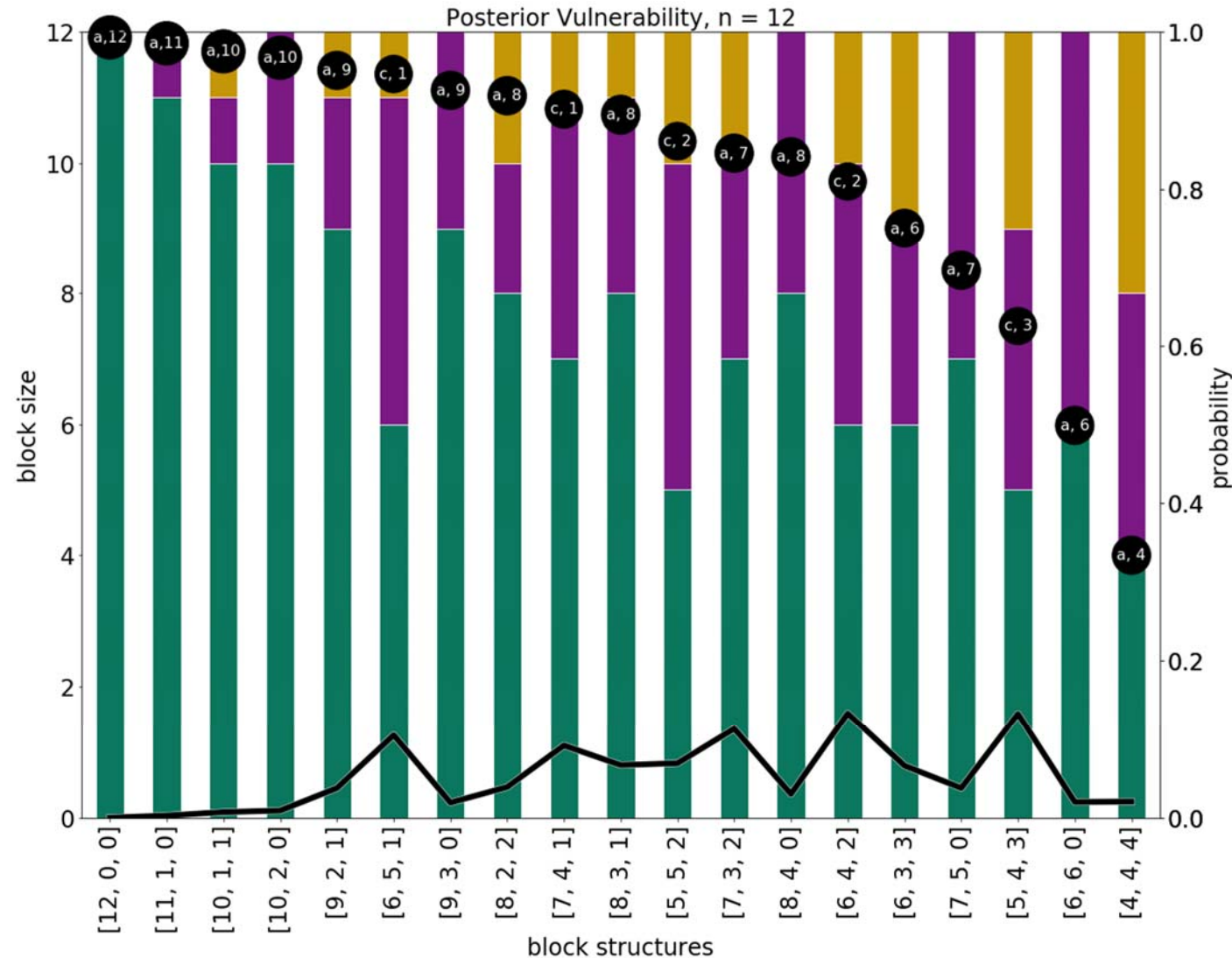
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- **Posterior vulnerability: ~0.813**



# Contributions



Revealed that there are scenarios where deterministic encryption is not safe even under a uniform prior.



Demonstrated a novel security framework by coupling the provable security approach of modern cryptography with QIF theory.



Illustrated that there is no one 'right' way to quantify leakage. Information flow depends on the operational scenario.